Numerical study of recent interior point approaches for linear programming

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Abstract

In this work, a comparative numerical study of a primal-dual interior point method based on recent descent directions is presented. We give a numerical appreciation of the numerical behavior of the considered algorithm taking into account the number of iterations as well as the time needed for optimality. The results obtained show the efficiency of these new directions.

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1 Introduction

Linear programming (LP) is certainly one of the greatest successes of operations research. Its importance lies, on the one hand, in the modelling power it offers despite the inherent limit imposed by the linearity of the functions involved. On the other hand, in the richness of the theory it has initiated and which has allowed the development of extremely efficient algorithms for its solution. Since its formulation and the development of the simplex method for its solution in the late 1940s, linear programming has remained the most widely used optimization model, which is certainly due to the robustness and stability of the available algorithms. The combinatorial nature of the simplex algorithm has probably caused LP to be considered for a long time as a separate class of problems from nonlinear programming.

Currently, primal-dual central path (CP) methods are the best interior point methods (IPMs) [1, 2, 7], because their algorithms admit good theoretical behavior and they are Newton-like which leads to numerical efficiency. The importance of CP for linear optimization has been recognized by several researchers, such as Megiddo [9] and Sonnevend [13].

In 2003, Darvay [3] was the first to introduce in his thesis a new approach based on the modification of the centrality equation:

$$xs = \mu e, \mu > 0.$$

Indeed, a real function of one variable $\varphi \in C^1$, $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ invertible, replaces the centrality equation by the following equivalent equation:

$$\varphi(xs) = \varphi(\mu e), \mu > 0.$$

Where $\mu > 0$ is a barrier parameter and $e \in \mathbb{R}^n$, with $e_i = 1$, $\forall i = 1, \ldots, n$. Then, through this modification, he obtained a large new family of Newton directions, such that:

$\varphi(t) = t$ gives the standard primal-dual algorithm.

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$\varphi(t) = t^2$ and $\varphi(t) = t^{\frac{q+1}{2}}$, where $q > 1$ give search directions differing from those defined by Peng et al. [11, 12] only by a constant multiplier.

Z. Darvay used the function $\varphi(t) = t^2$ in [5] to define a new primal-dual interior point algorithm.

The same author took in [4] the function $\varphi(t) = \sqrt{t}$ to present a new primal-dual interior point algorithm based on the appropriate search directions.

In this work, we are interested in the comparative numerical study of these different methods with the classical central trajectory method, to use their functions again in perspective.

The paper is organized as follows. In Section 2, we briefly describe the classical CP methods for LP and we give some new directions of CP, then we describe the algorithm prototype. In Section 3, we establish some numerical results. Finally, we conclude our work in Section 4.

2 Description of central path method for LP

We consider the LP problem in the following standard form:

$$(LP)\begin{cases} \min\ c^T x \\ Ax = b, \\ x \geq 0, \end{cases}$$

where $A \in \mathbb{R}^{m\times n}$, such that $rank(A) = m$, $b \in \mathbb{R}^m$ and $c \in \mathbb{R}^n$. The dual of the problem (LP) can be written in the following form:

$$(LD)\begin{cases} \max\ b^T y \\ A^T y + s = c, \\ s \geq 0,\ s \in \mathbb{R}^n, \\ y \in \mathbb{R}^m. \end{cases}$$

We denote by:

$F_{(LP)} = \{x \in \mathbb{R}^n : Ax = b,\ x \geq 0\}$, the set of feasible primal solutions of (LP).
$F^0_{(LP)} = \{x \in \mathbb{R}^n : Ax = b,\ x > 0\}$, the set of strictly feasible primal solutions of (LP).
$F_{(LD)} = \left\{y \in \mathbb{R}^m : A^T y + s = c,\ s \geq 0\right\}$, the set of feasible dual solutions of (LD).
$F^0_{(LD)} = \left\{y \in \mathbb{R}^m : A^T y + s = c,\ s > 0\right\}$, the set of strictly feasible dual solutions of (LD).
$F^0 = F^0_{(LP)} \times F^0_{(LD)}$, the set of strictly feasible primal-dual solutions of (LP) and (LD).

2.1 Classical central path methods

We associate the following penalized problem:

$$(LP)_\mu\begin{cases} \min\ f_\mu(x) \\ Ax = b, \\ x > 0, \end{cases}$$

where f_μ is the penalized function defined by:

$$f_\mu(x) = c^T x - \mu\sum_{i=1}^{n}\log(x_i),$$

and μ is a strictly positive barrier parameter.

Lemma 2.1. The function $f_\mu(x)$ is strictly convex.

Proof. the function $f_\mu(x)$ is written in the form:
$f_\mu(x) = c^T x - \mu \sum_{i=1}^{n} \log(x_i)$.
Indeed, $f_\mu(x) \in C^\infty$ and we have in particular:
$\nabla f_\mu(x) = c - \mu X^{-1} e$, with $e = (1, ..., 1)^T \in \mathbb{R}^n$,
$\nabla^2 f_\mu(x) = \mu X^{-2}$,
where $X = diag(x_1, ..., x_n)$ is a positive definite matrix, because $x_i > 0$, and $\mu > 0$, then $\nabla^2 f_\mu(x)$ is a positive definite matrix, hence the result. Q.E.D.

2.1.1 Properties of $f_\mu(x)$: see [6]

1. If $F^0_{(PL)}$ and $F^0_{(DL)}$ are non-empty, then for any $\mu > 0$, the problem $(PL)_\mu$ has a unique solution, denoted $x(\mu)$, and called the "central point".

2. When $\mu \to 0$, $x(\mu) \to x^*$ is an optimal solution of (LP).

3. The function $\mu \to (x(\mu), y(\mu), s(\mu))$ defines the central trajectory which we denote

$$T_C = \{(x(\mu), y(\mu), s(\mu)) : \mu > 0\}.$$

   T_C is called the central trajectory of $(PL)_\mu$.

4. $x(\mu)$ is uniquely defined by the following Karush-Khun-Tucker optimality conditions:

$$\begin{cases} c - \mu X^{-1} e - A^T y = 0, \\ Ax = b,\ x > 0, \end{cases}$$

where $y \in \mathbb{R}^m$ is the Lagrange multiplier associated with the constraint $Ax = b$ of the problem $(PL)_\mu$.
Let $s = \mu X^{-1} e \in \mathbb{R}^n_+$, the previous system becomes:

$$(S_\mu) \begin{cases} Ax = b,\ x > 0, \\ A^T y + s = c,\ s > 0, \\ Xs = \mu e,\ \mu > 0. \end{cases}$$

Note that (S_μ) corresponds to the complementarity conditions for a linear primal-dual program LP.

The system (S_μ) also denotes the optimality conditions for the following dual parametrized problem (LD):

$$(LD)_\mu \begin{cases} \max\ b^T y + \mu \sum_{i=1}^{n} \log(s_i) \\ A^T y + s = c, \\ s > 0. \end{cases}$$

Indeed, the (Karush-Kuhn-Tucker) optimality conditions for the problem $(LD)_\mu$ are given by:

$$(S\prime_\mu)\begin{cases} b - Ax = 0, \\ \mu S^{-1}e - x = 0, \\ A^T y + s = c, \end{cases}$$

where x is the Lagrange multiplier associated with the constraint $A^T y + s = c$ and $S = diag(s_1, ..., s_n)$. Hence, $(S\prime_\mu)$ is equivalent to (S_μ).

(S_μ) is a system of non-linear equations, for this reason, Newton's method is one of the most one used for its resolution.

At each μ, we find a solution $(x(\mu), y(\mu), s(\mu))$ close to the central trajectory (proximity condition).

Definition 2.2. The solution $(x(\mu), y(\mu), s(\mu))$ is said to be close to the central trajectory if it belongs to the set :

$T_C(\theta) = \{(x, y, s) \in F^0_{(LP)} \times F^0_{(LD)} \ / \ \|xs - \mu e\| \leq \theta\mu,\ 0 < \theta < 1\}$, where $\|.\|$ is the Euclidean norm.

The system (S_μ), can be written as:

$$F(x, y, s) = 0,$$

where

$$F(x, y, s) = \begin{pmatrix} XSe - \mu e \\ Ax - b \\ A^T y + s - c \end{pmatrix}$$

Newton's iteration is defined by:

$$(x_+, y_+, s_+) = (x, y, s) + (\Delta x, \Delta y, \Delta s),$$

where $(\Delta x, \Delta y, \Delta s)$ is the solution of the linear system:

$$\begin{pmatrix} S & 0 & X \\ A & 0 & 0 \\ 0 & A^T & I \end{pmatrix} \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta s \end{pmatrix} = - \begin{pmatrix} XSe - \mu e \\ 0 \\ 0 \end{pmatrix} \tag{LS}$$

Theoretically, we assume that the initial primal-dual solution is strictly feasible, and close to the central trajectory.

2.2 Recent descent directions of CP methods

In 2003, Z. Darvay [3] defined a new method to find search directions for interior point methods (IPMs). The main idea of this method is defined as follows:

Finding the optimal solutions of (LP) and (LD) is equivalent to solve the following system:

$$\begin{cases} Ax = b,\ x \geq 0, \\ A^T y + s = c,\ s \geq 0, \\ xs = 0, \end{cases} \tag{1}$$

where xs is the product in coordinates of the vectors x and s, that is

$$xs = [x_1 s_1, ..., x_n s_n] \in \mathbb{R}^n.$$

We will also use the notation

$$\tfrac{x}{s} = [\tfrac{x_1}{s_1}, ..., \tfrac{x_n}{s_n}] \in \mathbb{R}^n.$$

The main idea of the interior point methods (IPMs) is to replace the third equation of the system (1), called the complementarity condition, by the parametric equation $xs = \mu e$, with $\mu > 0$ and $e = (1, 1, \ldots, 1)^T \in \mathbb{R}^n$. Then, we obtain the following system:

$$\begin{cases} Ax = b,\ x \geq 0, \\ A^T y + s = c,\ s \geq 0, \\ xs = \mu e,\ \mu > 0. \end{cases} \tag{2}$$

If the interior point condition (IPC) holds, that is to say, there exists (x_0, y_0, s_0) such that:

$$Ax_0 = b,\ x_0 > 0,\ A^T y_0 + s_0 = c,\ s_0 > 0.$$

Then for a fixed $\mu > 0$, the system (2) has a unique solution, called the μ-center (Sonnevend [13]). The set of μ-centers for $\mu > 0$ forms a well-conformed curve, called the central path.

From the last equation of system (2) we have $x_i(\mu)s_i(\mu) = \mu$, $\forall i = 1, ..., n$, then $x^T s = n\mu$ where $x^T s$ denotes the duality jump, so it suffices to find (approximately) all points on the central trajectory when $\mu \to 0$.

The principle of Z. Darvay's method is to replace the centrality equation:

$$\tfrac{xs}{\mu} = e,$$

in system (2) by the following new equation:

$$\varphi\left(\frac{xs}{\mu}\right) = \varphi(e).$$

Where φ is a invertible function, that is to say φ^{-1} exists.

The system (2) becomes as follow:

$$\begin{cases} Ax = b,\ x \geq 0, \\ A^T y + s = c,\ s \geq 0, \\ \varphi\left(\frac{xs}{\mu}\right) = \varphi(e),\ \mu > 0. \end{cases} \tag{3}$$

We apply Newton's method to the nonlinear system (3), we obtain:

$$\begin{cases} A\Delta x = 0, \\ A^T \Delta y + \Delta s = 0, \\ \frac{s}{\mu}\varphi'(\frac{xs}{\mu})\Delta x + \frac{x}{\mu}\varphi'(\frac{xs}{\mu})\Delta s = \varphi(e) - \varphi(\frac{xs}{\mu}). \end{cases} \tag{4}$$

We introduce the vector: $v = \sqrt{\frac{xs}{\mu}}$.

We also introduce the following notations: $dx = \frac{v\Delta x}{x}$, $ds = \frac{v\Delta s}{s}$.
We can obtain easily: $\mu v(dx + ds) = s\Delta x + x\Delta s$ and $dxds = \frac{\Delta x \Delta s}{\mu}$.
Therefore, the linear system (4) can be written in the following form:

$$\begin{cases} \bar{A}dx = 0, \\ \bar{A}^T\Delta y + ds = 0, \\ dx + ds = pv, \end{cases} \tag{5}$$

where: $pv = \frac{\varphi(e)-\varphi(v^2)}{v\varphi'(v^2)}$, and $\bar{A} = \frac{1}{\mu}Adiag(\frac{x}{v})$.

We mention that $\varphi(t) = t$ gives $pv = v^{-1} - v$, which corresponds to the standard primal-dual algorithm CP with $O(\sqrt{n}\log\frac{n}{\varepsilon})$-iteration complexity. Recently, Peng et al [12] observed that a new search direction can be obtained by taking $pv = v^{-3} - v$. The same authors analyzed in [11] the case $pv = v^{-q} - v$, where $q > 1$. They also introduced a class of search directions based on self-regular approximates (Peng et al [10]).
The general approach of Z. Darvay can be particularized in order to obtain the directions defined in [11] and [12]. For $\varphi(t) = t^2$ we get $pv = \frac{1}{2}(v^{-3} - v)$, and for $\varphi(t) = t^{\frac{q+1}{2}}$, where $q > 1$ we get $pv = \frac{2}{q+1}(v^{-q} - v)$. We conclude that these search directions do not differ from those defined in [11, 12] than by a constant multiplier.

In other hand, Z. Darvay takes the function $\varphi(t) = \sqrt{t}$ to present a new model of primal-dual interior point algorithm based on the appropriate search directions, with $O(\sqrt{n}\log\frac{n}{\varepsilon})$-iteration complexity, and $pv = 2(e - v)$.

We define a proximity measure of the central path

$$\delta(xs, \mu) = \frac{\|pv\|}{2} = \|e - v\| = \left\|e - \sqrt{\frac{xs}{\mu}}\right\|,$$

Let's pose $qv = dx - ds$.

Note that according to (5), we have $(dx)^T ds = 0$, so the vectors dx and ds are orthogonal, which implies

$$\|pv\| = \|qv\|.$$

Consequently, we mention that the proximity measure can also be expressed with the vector qv, so

$$\delta(xs, \mu) = \frac{\|qv\|}{2},$$

then

$$dx = \frac{pv+qv}{2} \text{ and } ds = \frac{pv-qv}{2}$$

from where

$$dxds = \frac{pv^2-qv^2}{4}.$$

We are now ready to describe the algorithm of these methods.

2.3 Description of algorithm

The generic representation of this algorithm is given as follows:

Begin algorithm

Let $\varepsilon > 0$ be the accuracy parameter, $0 < \theta < 1$ the update parameter (default $\theta = \frac{1}{2\sqrt{n}}$) and $0 < \tau < 1$ the proximity parameter (default $\tau = \frac{1}{2}$), $k = 0$.

Suppose that for the triple (x_0, y_0, s_0) the interior point condition holds and let $\mu_0 = \frac{x_0^T s_0}{n}$. Furthermore, suppose that $\delta(x_0 s_0, \mu_0) < \tau$.

While $x^T s > \varepsilon$ do
$-\mu = (1-\theta)\mu$,
$-$Substitute $\varphi(t)$ into (4) and compute (Δx, Δy, Δs),
$-$Take $x = x + \Delta x$, $y = y + \Delta y$, $s = s + \Delta s$,
$-$Pose $k = k + 1$.
end while
end algorithm.

3 Numerical experiments

We present in this section a comparative tests with different examples taken from the literature [8]. We take $\varepsilon = 10^{-4}$.

3.1 Examples with fixed size

Example 1

$A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & -3 \end{pmatrix}$, $c = \begin{pmatrix} 1 & 2 & 3 & 4 \end{pmatrix}^T$, $b = \begin{pmatrix} 1 & 0.5 \end{pmatrix}^T$.

Where : $x_0 = \begin{pmatrix} 0.5 & 0.27 & 0.14 & 0.09 \end{pmatrix}^T$, $s_0 = \begin{pmatrix} 1 & 2 & 3 & 4 \end{pmatrix}^T$, $y_0 = \begin{pmatrix} 0 & 0 \end{pmatrix}^T$.

Example 2

$A = \begin{pmatrix} 2 & 1 & 1 & 0 & 0 \\ 1 & 2 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{pmatrix}$, $c = \begin{pmatrix} -5 & -5 & 0 & 0 & 0 \end{pmatrix}^T$, $b = \begin{pmatrix} 8 & 7 & 3 \end{pmatrix}^T$.

Where : $x_0 = \begin{pmatrix} 2.2534 & 1.5743 & 1.9185 & 1.5976 & 1.4256 \end{pmatrix}^T$,
$s_0 = \begin{pmatrix} 1 & 3 & 2 & 2 & 2 \end{pmatrix}^T$, $y_0 = \begin{pmatrix} -2 & -2 & -2 \end{pmatrix}^T$.

Example 3

$$A = \begin{pmatrix} 1 & 0 & -4 & 3 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 5 & 3 & 1 & 0 & -1 & 3 & 0 & 1 & 0 & 0 & 0 & 0 \\ 4 & 5 & -3 & 3 & -4 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 2 & 1 & -5 & 0 & 0 & 0 & 1 & 0 & 0 \\ -2 & 1 & 1 & 1 & 2 & 2 & 0 & 0 & 0 & 0 & 1 & 0 \\ 2 & -3 & 2 & -1 & 4 & 5 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$c = \begin{pmatrix} -9 & -4 & 4 & -7 & -2 & -6 & 0 & 0 & 0 & 0 & 0 & -4 \end{pmatrix}^T$,
$b = \begin{pmatrix} 7 & 12 & 7 & -2 & 6 & 10 \end{pmatrix}^T$.

Where : $x_0 = s_0 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}^T$,
$y_0 = \begin{pmatrix} -1 & -1 & -1 & -1 & -1 & -1 \end{pmatrix}^T$.

3.2 Example with variable size

Example 4 (Cube problem): $n = 2m$,

$$A[i,j] = \begin{cases} 1 & \text{if } j = i \text{ or } j = i + m \\ 0 & \text{otherwise} \end{cases}, \; c[i] = \begin{cases} -1 & \text{if } i = 1, ..., m \\ 0 & \text{if } i = m+1, ..., n \end{cases},$$

$b[j] = 2$ for $j = 1, ..., m$

Where

$$x_0[i] = 1, \text{ for } i = 1, ..., n, \; s_0[i] = \begin{cases} 1 & \text{if } i = 1, ..., m \\ 2 & \text{if } i = m+1, ..., n \end{cases},$$

$y_0[j] = -2$ for $j = 1, ..., m$

We denote by:

- M1, the classical central trajectory interior point method.
- M2, the central trajectory interior point based on Newton's direction with the choice of $\varphi(t) = \sqrt{t}$.
- M3, the central trajectory interior point based on Newton's direction with the choice of $\varphi(t) = t^2$.
- M4, the central trajectory interior point based on Newton's direction with the choice of $\varphi(t) = t^{\frac{q+1}{2}}$($q = 2$).
- M5, the central trajectory interior point based on Newton's direction with the choice of $\varphi(t) = t^{\frac{q+1}{2}}$($q = 4$).
- $T(s)$, the execution time necessary for optimally.
- k, the number of iterations.
- Z^*, the optimal value.

The obtained results for the three fixed size examples are presented in the following comparative table:

	Example 1	**Example 2**	**Example 3**
M1	$T(s) = 0.012732$ $Z^* = 1.375038$ $k = 35$	$T(s) = 0.031686$ $Z^* = -24.998630$ $k = 48$	$T(s) = 0.058922$ $Z^* = -28.815228$ $k = 76$
M2	$T(s) = 0.012420$ $Z^* = 1.375037$ $k = 35$	$T(s) = 0.034147$ $Z^* = -24.998631$ $k = 48$	$T(s) = 0.017488$ $Z^* = -28.815227$ $k = 75$
M3	$T(s) = 0.011713$ $Z^* = 1.375040$ $k = 35$	$T(s) = 0.014473$ $Z^* = -24.998629$ $k = 48$	$T(s) = 0.017432$ $Z^* = -28.815228$ $k = 76$
M4	$T(s) = 0.012125$ $Z^* = 1.375039$ $k = 35$	$T(s) = 0.011704$ $Z^* = -24.998630$ $k = 48$	$T(s) = 0.018122$ $Z^* = -28.815228$ $k = 76$
M5	$T(s) = 0.012389$ $Z^* = 1.375042$ $k = 35$	$T(s) = 0.015541$ $Z^* = -24.998628$ $k = 48$	$T(s) = 0.018326$ $Z^* = -28.815228$ $k = 76$

The following tables summarize the results of Example 4 for different sizes:

	$(m, n) = (10, 20)$	$(m, n) = (25, 50)$
M1	$T(s) = 0.030049$ $Z^* = -19.999953$ $k = 107$	$T(s) = 0.128903$ $Z^* = -49.999951$ $k = 185$
M2	$T(s) = 0.029993$ $Z^* = -19.999954$ $k = 107$	$T(s) = 0.121438$ $Z^* = -49.999952$ $k = 185$
M3	$T(s) = 0.020260$ $Z^* = -19.999953$ $k = 107$	$T(s) = 0.108776$ $Z^* = -49.999951$ $k = 185$
M4	$T(s) = 0.029889$ $Z^* = -19.999953$ $k = 107$	$T(s) = 0.114278$ $Z^* = -49.999951$ $k = 185$
M5	$T(s) = 0.030529$ $Z^* = -19.999853$ $k = 107$	$T(s) = 0.110847$ $Z^* = -49.999951$ $k = 185$

	$(m,n)=(100,200)$	$(m,n)=(500,1000)$
M1	$T(s)=4.652430$ $Z^*=-1.99999950\times10^2$ $k=415$	$T(s)=945.795299$ $Z^*=-9.99999950\times10^2$ $k=1037$
M2	$T(s)=4.557001$ $Z^*=-1.99999951\times10^2$ $k=415$	$T(s)=1012.359368$ $Z^*=-9.99999950\times10^2$ $k=1037$
M3	$T(s)=4.141557$ $Z^*=-1.99999951e\times10^2$ $k=415$	$T(s)=921.400917$ $Z^*=-9.99999950\times10^2$ $k=1037$
M4	$T(s)=4.471705$ $Z^*=-1.99999951\times10^2$ $k=415$	$T(s)=941.752978$ $Z^*=-9.99999950\times10^2$ $k=1037$
M5	$T(s)=4.647445$ $Z^*=-1.99999951\times10^2$ $k=415$	$T(s)=923.948862$ $Z^*=-9.99999950\times10^2$ $k=1037$

Comments

The realized numerical simulations show that:

• The central trajectory interior point algorithm gives the same number of iterations with different choices of $\varphi(t)$. On the other hand, the solution computation time with the choice of $\varphi(t)=t^2$ is generally better than the other choices. Similarly, for the optimal value the choice of $\varphi(t)=\sqrt{t}$ is generally better than the others.

• Generally, the Newton direction based central trajectory interior point algorithm with the new choices of $\varphi(t)$ gives better results than with the standard primal-dual algorithm ($\varphi(t)=t$).

4 Conclusion

In this work we have considered a recents class of search directions based on equivalent form of the central path. The main idea was that Z. Darvay introduced a function φ, and applied Newton's method for the new system. According the choice of the function φ, we obtained the directions defined in [11, 12]. Z. Darvay used $\varphi(t)=\sqrt{t}$ to present a new model of primal-dual interior point method. Our established numerical tests, shown that generally the new choices of $\varphi(t)$ given better results than the standard primal-dual interior point method.

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